Lotteries, certainty equivalent and risk premium

An individual with a utility function $U(x) = 2x^2$ must choose between the following options:

- Play a lottery with a prize of \$0 with a probability of 50% and \$100 with a probability of 50%.
- Play a lottery with a prize of \$20 with a probability of 60% and \$60 with a probability of 40%.

Answer the following questions:

1. Determine the expected value of lottery A
2. Determine the expected utility of both options
3. Find the value of Z such that the individual is indifferent between choosing Z and option A (certainty equivalent).
4. Calculate the risk premium and explain it.

Solution

1. $$VE(A) = 0 \times 0.5 + 100 \times 0.5 = 50$$

2. $$UE(A) = U(0) \times 0.5 + U(100) \times 0.5 = 0 + 20,000 \times 0.5 = 10,000$$

   $$UE(B) = U(20) \times 0.6 + U(60) \times 0.4 = 800 \times 0.6 + 7,200 \times 0.4 = 3,360$$

3. $$10,000 = 2z^2$$

   $$5,000 = z^2$$

   $$\sqrt{5,000} = z$$

   $$z = 70.71$$

4. Calculate the risk premium

   The risk premium is the amount an individual would pay to avoid taking a risk. It can be calculated as the difference between the expected value of the lottery and the certainty equivalent (CE).

   The risk premium (RP) is:

   $$RP = VE(A) - CE = 50 - 70.71 \approx -20.71$$

   A negative risk premium indicates that the individual prefers taking the risk of the lottery over receiving a certain amount equivalent to the expected value of the lottery. This could be due to the individual's risk-seeking behavior or their perception that the potential high payoff from the lottery is more attractive than the certainty of a smaller amount. For the individual to choose a certain amount we need to give him $20.71.